Amplitude–Frequency Characteristics of Polymer Electro-Magnetic Dynamic Tri-Screw Extrusion

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ABSTRACT: The mechanism of plastic forming and processing in electromagnetic dynamic tri-screw extruder is very sophisticated and the investigation of amplitude–frequency characteristic acts as the foundation of equipment design and the optimization of polymer processing parameters. A mathematical and analytical model of plastic forming in such extruder was developed and the results were nondimensional-normalized. To validate the mathematical solutions experiments based on LDPE were carried out and the experimental vibration amplitude and vibration frequency curve was obtained. Three conclusions can be drawn herein: (1) the experimental results hold a good agreement with the calculations, and thus the feasibility of the proposed model is validated; (2) the possibility of resonance closely relates to polymer melt viscosity, rotating speed, and geometry parameters of the screw; (3) resonance of the tri-screw extruder is seldom observed under normal conditions and there exists an inverse correlationship between vibration frequency and amplitude. © 2011 Wiley Periodicals, Inc. J Appl Polym Sci 122: 1778–1784, 2011

Key words: tri-screw extruder; amplitude–frequency characteristic; electromagnetic; vibration force field

INTRODUCTION

The introduction of axial electromagnetic vibration force field into polymer processing was firstly developed in 1980s. Fridman et al.[1] applied low frequency mechanical vibration at the die with a partial eccentric mechanism. They extruded Polypropylene at the vibration amplitudes of 4.8°, 11.5°, and 22.3°, respectively, with the identical vibration frequency of 25 Hz. The corresponding shear strains of eccentric stick were observed as 75%, 180%, and 335%. Subsequently, Isayev et al.[2] introduced ultrasonic field into polymer processing. Compared with lower frequency mechanical vibration field, ultrasonic field could stir the medium and produce cavity without any mechanical noise, which could further promote chemical–physical reaction. Therefore, it had advanced an extensive application in polymer processing. Qu et al.[3] introduced pulsating electromagnetic force field to polymer processing to produce periodical mechanical vibration and realized the coupling of heat and mechanical fields. Finally, they invented an electronic-magnetic dynamic plasticizing extruder and injector.[4] The equipment and method has obtained a wide recognition in academic and industrial circles and achieved excellent economical and social benefits for its outstanding performance. Qu et al.[3] proposed an electromagnetic single screw extruder and established a model to investigate the polymer melting process.

Many researches[3,5–10] have been carried out to explore the effects and mechanism of such introduced vibration force field in polymer forming and processing. Zeng et al.[6] developed a molecular scale

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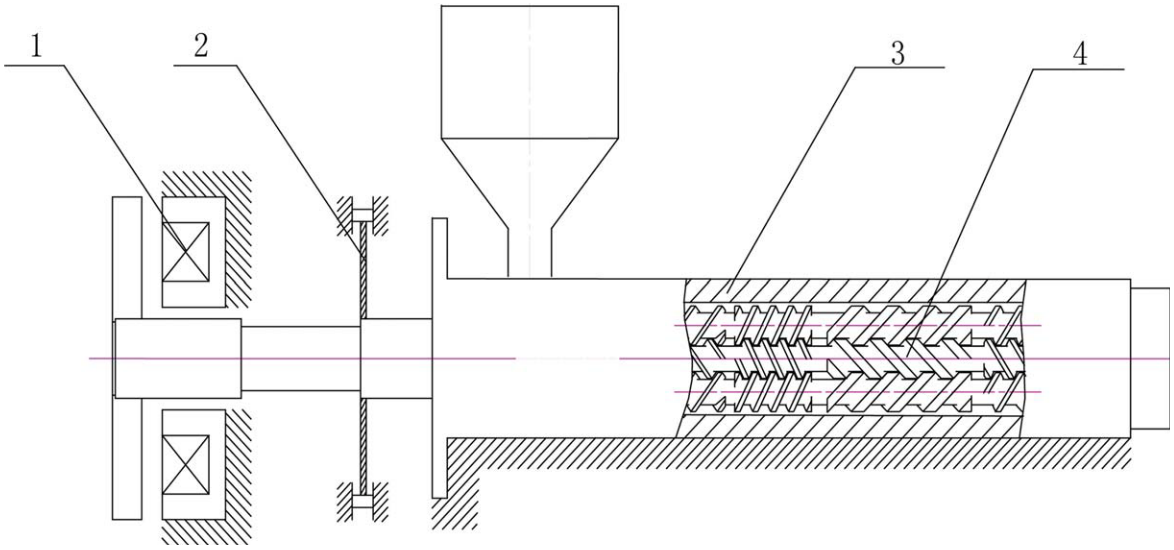

Figure 1 Sketch of polymer dynamic tri-screw extruder with middle screw vibrating (1 exciter, 2 pulsation transfer plate, 3 barrel, and 4 tri-screw). [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

model to describe the plasticizing mechanism for polymer under the impact of axial vibration force field. Further the melting capacity and viscosity of polymer melts subject to vibration force field was also explored by Dr. Zeng[9] and the relationship with vibration amplitude and frequency was also presented. All the aforementioned studies show that both vibration amplitude and frequency have significant influence on polymer processing, namely the superimposed axial vibration force field introduced in extruder can improve the polymer products quality and lower the energy consumption.

However, the equivalent relationship between vibration amplitude and vibration frequency has seldom been investigated, especially for tri-screw extruder. Qu et al.[3] established a mechanical model to analyze the axial force distribution of the screw under vibration force field and produced some amplitude–frequency characteristic curves for polymer plasticizing in the single screw superimposed with an axial vibration force. Besides, the authors carried out a series experiments and obtained a good agreement between the theoretical and experimental results. The model developed by Qu is too simple to characterize the amplitude–frequency characteristics of polymer in tri-screw electromagnetic dynamic extruder proposed herein. While the amplitude–frequency characteristics play a significant role in optimizing the vibration parameter for different kind of polymer or polymer composites. Eventually, it can improve the application of vibration force field in plastic dynamic processing and offer theoretical reference for exploring the plasticizing mechanism of polymer and designing plastic dynamic molding equipments.

An electromagnetic dynamic tri-screw extruder was developed by introducing superimposed axial vibration force field into the whole polymer extrusion process. To explore the extrusion characteristics of polymer in such extruder and optimizing the vibration parameters in plastic molding, a mathematical and analytical model was developed, and some experiments with LDPE were carried out for the assessment of the proposed theory.

Mathematical and analytical model

Damping

Electromagnetic dynamic tri-screw extruder is a complex structure and its vibration characteristics remain difficult to calculate. As illustrated in Figure 1, the Electromagnetic dynamic tri-screw extruder is composed of three parallel screws and the axial line of which lay on the same line. The middle screw and the side one can form a corotating twin screw extruder. Besides, the middle one can vibrate while rotating. The rotating speed of the three screws keeps identical. Some simplifications and assumptions should be made for convenience before developing the physical and mathematical model. First, the side screws keep stationary, whereas the middle one vibrates (Fig. 1). Second, the screws are considered to be rigid as concentrated mass with no available deformation. Again, we propose no direct contact exists between screws and barrel. Then, the damping of extruder can be divided into four parts including pressure damping between screw flights among vibrating one and nonvibrating ones, viscous damping between the tops of vibrating and nonvibrating screw flights,

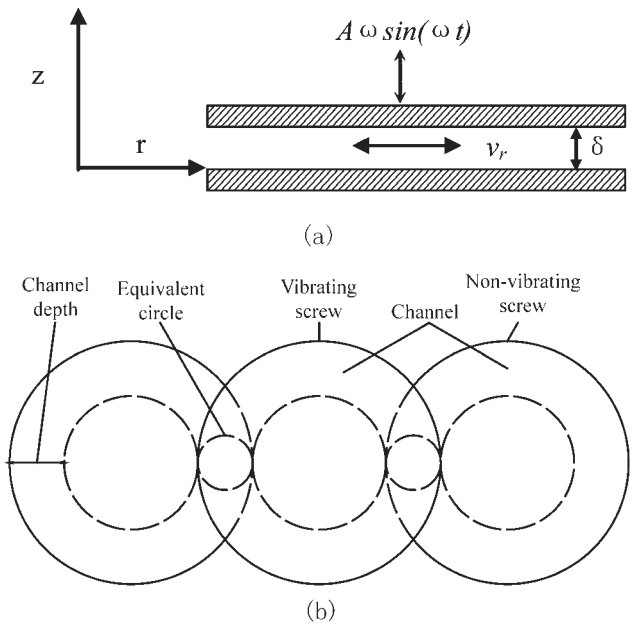

Figure 2 (a) Model of adjacent screw flights and (b) simplification of the acting force area.

viscous damping between bottoms of vibration screw channel and barrel wall, and viscous damping between the top of screw flights and barrel wall.

Resistance of polymer melts in side gap of screw flight

The interaction between adjacent screw flights can be simplified as illustrated in Figure 2(a). Two plates represent the screw flights and the acting surface between two screw flights can be equivalent to a circle with radius of $R = H/2$ and H represents screw channel depth as illustrated in Figure 2(b). Propose the vibrating speed of the middle screw is $A\omega\sin\omega t$, in which A represents vibration amplitude ω represents vibration frequency. The polymer melt between two screw flights can diffuse in the radial direction with the velocity of v_r under vibration force field. The velocity induced by vibration force in the z direction can be written as follows:

$$v_z = A\omega\cos\omega t \tag{1}$$

where A and ω is the vibration amplitude and frequency of the middle screw respectively, t denotes time corresponding to vibration. We can obtain from the aspect of mass conservation law.

$$\pi r^2 v_z = 2\pi r\delta\overline{v_r} \tag{2}$$

$$\overline{v_r} = \frac{rv_z}{2\delta} = \frac{1}{\delta}\int_0^{\delta} v_r dz \tag{3}$$

where $\overline{v}_r$ is the mean velocity in the r direction and δ denotes the gap between two screw flights. The polymer melt flowing between is supposed to be Newtonian fluid, and therefore, we have the following motion equation:

$$\frac{dP}{dr} = \eta\frac{d^2v_r}{dz^2} \tag{4}$$

Boundary conditions

$$v_r(0) = 0, \quad v_r(\delta) = 0 \tag{5}$$

Herein, P is the pressure distribution and η is the viscosity of the polymer. Solving above equations, we can obtain as follows:

$$v_r = \frac{1}{2\eta}\frac{dP}{dr}(z-\delta)z \tag{6}$$

Substituting eq. (6) into eq. (4), we have the following:

$$\frac{rv_z}{2\delta} = \frac{1}{2\eta\delta}\frac{dP}{dr}\int_0^{\delta}\left(z^2 - z\delta\right)dz \tag{7}$$

By integrating eq. (7) the following form is obtained.

$$rv_z + \frac{\delta^3}{6\eta}\frac{dP}{dr} = 0 \tag{8}$$

Again solving eq. (8), we have the following:

$$P(r) = P_0 + \frac{3\eta v_z\left(R^2 - r^2\right)}{\delta^3} \tag{9}$$

where R is the major radius of the equivalent circle.

Thus, the resistance induced by melt pressure between screw flights is obtained.

$$F_1 = \frac{\pi n_e P_0 H^2}{4} + \frac{3\pi\eta A\omega H^4 n_e\cos\omega t}{64\delta^3} \tag{10}$$

where P_0 is the pressure of polymer melt within barrel, n_e represents the pitch number of screw, and H is the depth of the screw channel.

Viscous resistance of polymer melts between screw flight and barrel wall

Simplified model (Fig. 3) can be established to present the melt flow between screw flight and barrel wall, where the upper plate represents barrel and the lower stationary one represents screw flight. We assume that the barrel vibrates with the velocity of $A\omega\sin\omega t$, whereas the other two screws keep stationary. The

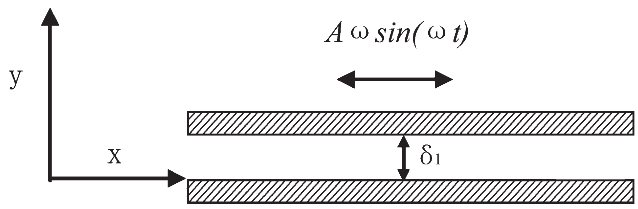

Figure 3 Model of screw flight and barrel.

melt velocity distribution between the two plates is v_x, and δ_1 represents the gap between the barrel wall and the screw flight. The constitutive equation can be expressed in the below form

$$\tau_{xy} = \eta \frac{dv_x}{dy} = \eta \frac{A\omega \cos \omega t}{\delta_1} \tag{11}$$

Then, we have the viscous resistance

$$F_2 = S_2 \eta A\omega \cos \omega t / \delta_1 \tag{12}$$

where S_2 is the total area on the top of vibrating screw flight.

Damping factor

Similar with above two subsections, we can calculate out the viscous resistance between the bottom of vibrating screw channel and barrel wall as well as that between the bottom of vibrating screw channel and the top of screw flights in the following equation.

$$F_3 = S_3 \eta A\omega \cos \omega t / \delta_3 \tag{13}$$

$$F_4 = S_4 \eta A\omega \cos \omega t / \delta_4 \tag{14}$$

where δ_3, δ_4 denotes the gap between the bottom of vibrating screw channel and barrel wall together with that between the bottom of vibrating screw channel and the top of nonvibrating screw flights respectively. S_3, S_4 refers to the acting areas corresponding to aforementioned force. Then, the total resistance can be written as follows:

$$\begin{aligned} F &= F_1 + F_2 + F_3 + F_4 \\ &= \pi n_e P_0 H^2/8 + (3\pi\eta n_e H^4/64\delta^3 + S_2\eta/\delta_1 \\ &\quad + S_3\eta/\delta_3 + S_2\eta/\delta_4) A\omega \cos \omega t \end{aligned} \tag{15}$$

Thus, the damping factor can be obtained

$$\beta = (3\pi\eta H^4/64\delta^3 + S_2\eta/\delta_1 + S_3\eta/\delta_3 + S_4\eta/\delta_4)/2m \tag{16}$$

where m is the general mass of the polymer system.

Exciting force

Specially designed solenoid exciter was adopted in the electromagnetic dynamic tri-screw extruder. The input current behaves as cosinoidal (or sinoidal) half wave as displayed in Figure 4 with the current peak of I_m.

Exciting force produced by solenoid exciter can be written as follows:

$$F_1(t) = F_a\left[\frac{3}{2} + 2\sin\left(\omega t - \frac{\pi}{2}\right)\right] \tag{17}$$

where F_a is fundamental electromagnetic force, and it further can be rewritten as follows:

$$F_a = \frac{B_a^2 S}{\mu_0} = \frac{(1-\sigma_a)^2 L_0^2 I_m^2}{N^2 \mu_0 S} \sin^2 \psi \tag{18}$$

where B_a represents magnetic flux density without considering variation of inductance coefficient σ_a; ψ denotes the phase angle of magnetic flux density; L_0 represents the total inductance in the current when the mean air gap remains δ_0; μ_0 is the permeability in vacuum; N is the number of coil turns; S denotes sectional area of single coil.

The exciting force varies with time cosinoidally (or sinoidally), the frequency of which is the same as that of input current of the exciter controller. $2F_a$ is the amplitude of exciting force and $\frac{3}{2}F_a$ is the balance location of electromagnetic exciting force, which is determined by the material, structure, dimensions, and the current peak I_m of exciter controller. The amplitude $2F_a$ of electromagnetic exciting force F_1
(t) and frequency ω can be regulated via adjusting the input current amplitude and frequency of exciter controller when the material, structure, and dimensions are determined.

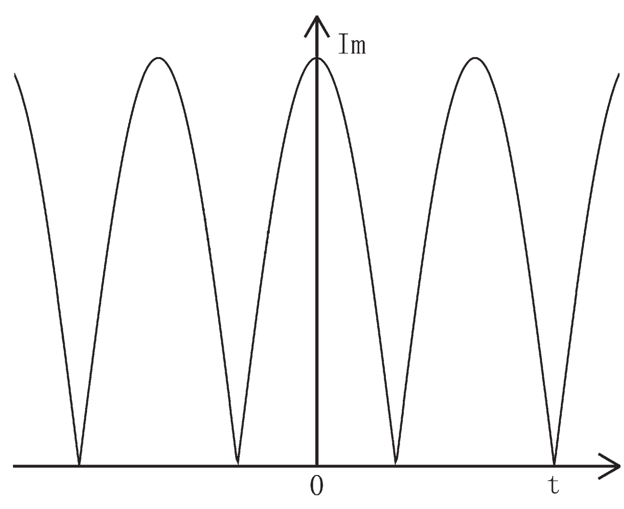

Figure 4 Current output wave of solenoid exciter power.

Amplitude–frequency characteristic

The forced vibration equation of the middle vibrating screw can be written as follows:

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = F_1(t) \quad (19)$$

where $x, \dot{x}, \ddot{x}$ is the vibration displacement, vibration velocity, and vibration acceleration of the vibrating screw.

Substituting eqs. (15), (16), and (17) into eq. (19), the following motion equation can be obtained.

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = F_a\left[\frac{3}{2} + 2\sin\left(\omega t - \frac{\pi}{2}\right)\right]/m - \pi n P_0 H^2/(8m) - \pi D^2 P_1/(4m) \quad (20)$$

where $\omega_0 = \sqrt{k/m}$ is the resonant frequency of the polymer system, k is the rigidity of the polymer system, and D is the diameter of the screw. Solving above equation, we can obtain vibration displacement of the vibrating screw.

$$x = A_1 \exp(-\beta t)\cos(\omega_0 t + \alpha) + A_2\cos(\omega t + \varphi) + \frac{12F_a - \pi n_e P_0 H^2 - 2\pi D^2 P_1}{8m\omega_0^2} \quad (21)$$

The first term of above equation is damping attenuation, which equals to zero when time tends to be infinity, and thus it can be ignored. The second term is steady vibration part under compelling force.

$$A_2 = 2F_a/\left[m\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + 4\beta^2\omega^2}\right] \quad (22)$$

$$\tan\varphi = -2\beta\omega/\left(\omega_0^2 - \omega^2\right) \quad (23)$$

Substituting all the coefficients of eqs. (22) and (23) into eq. (21), we have the following:

$$x = \frac{2F_a\cos\left[\omega t - \arctan\left(\frac{2\beta\omega}{\omega_0^2 - \omega^2}\right)\right]}{m\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + 4[(3\pi\eta n_e H^4/64\delta^3 + S_2\eta/\delta_2 + S_3\eta/\delta_3 + S_4\eta/\delta_4)/2m]^2\omega^2}} + \frac{12F_a - \pi n_e P_0 H^2 - 2\pi D^2 P_1}{8m\omega_0^2} \quad (24)$$

Equation (24) is a cosinoidal vibration equation, where ω is the vibration frequency, φ is the phase angle, and A denotes vibration amplitude. The balance location can be calculated as follows:

$$x_0 = \frac{12F_a - \pi n_e P_0 H^2 - 2\pi D^2 P_1}{8m\omega_0^2} \quad (25)$$

When $t = 0$, the maximum value of x can be obtained.

$$A = X - x_0 = \frac{2F_a}{m\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + 4[(3\pi\eta n_e H^4/64\delta^3 + S_2\eta/\delta_1 + S_3\eta/\delta_3 + S_4\eta/\delta_4)/2m]^2\omega^2}} \quad (26)$$

where X is the maximum displacement. Further letting

$$k_1 = (3\pi\eta n_e H^4/64\delta^3 + S_2\eta/\delta_1 + S_3\eta/\delta_3 + S_4\eta/\delta_4)/(2m) \quad (27)$$

Thus, eq. (26) can be rewritten as follows:

$$A = \frac{2F_a}{m\omega_0^2\sqrt{\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2 + \frac{4k_1^2\omega^2}{\omega_0^4}}} \quad (28)$$

Letting

$$A^* = Am\omega_0^2/F_a \quad (29)$$

$$\omega^* = \omega/\omega_0 \quad (30)$$

$$k_1^* = k_1/\omega_0 \quad (31)$$

A dimensionless amplitude–frequency characteristic expression can be obtained

$$A^* = \frac{2}{\sqrt{\left(1 - \omega^{*2}\right)^2 + 4k_1{}^{*2}\omega^{*2}}} \quad (32)$$

EXPERIMENTAL

Materials

LDPE was bought from Guangdong Maoming petrochemical Ltd (China). with an extrusion grade marked 951-050 was chosen as the testing polymer. To simplify the calculation, the viscosity of the LDPE melt was determined by rheological experiments and assumed to be constant of 10^3 Pa s under the experimental extrusion condition (temperature of 175°C and

TABLE I
Parameters of the Three Identical Screws

D (mm)	H (mm)	n_e	δ (mm)	δ_1 (mm)	δ_3 (mm)	δ_4 (mm)	S_2 (mm^2)	S_3 (mm^2)	S_4 (mm^2)
20	3.2	21	7.5	1	4.2	1	2638.86	15252.6	2638.86

rotation speed of 60 rpm). The melt density of the LDPE is 810 kg/m^3. The resonant frequency is obtained by dynamic rheological tests and the value of resonant frequency remains at $\omega_0 = 24$ Hz.

Apparatus

Electromagnetic dynamic tri-screw extruder equipped with a vibration exciter was adopted to extrude the polymer LDPE. The structure parameters of the three identical screws are listed in Table I. FLUKE-43 power quality analyzer made by Fluke in USA was used to monitor and record the power consumed by the whole extruder.

Procedures

Step.1 Install the Fluke power quality analyzer in the extruder main circuit and start the extruder heating system along with the water cooling system.

Step.2 Add LDPE pallets into the hopper of the extruder and start the motors which are used to driven the screw to rotate in the barrel and in the hopper when the temperature reaches the preset value.

Step.3 Adjust the speed of the screws so that the polymer melt can be fully plasticized and record the power value the extruder consumed and the speed of the screw as well as the polymer feeding speed.

Step.4 Keep the same speed of the screw and the polymer feeding speed, start the vibration exciter with different vibration amplitude (or vibration frequency), and adjust the corresponding vibration frequency (or vibration amplitude) to keep the energy power as a constant of the above record. Then, collect such vibration amplitude and vibration frequency combinations, from which the vibration amplitude and vibration frequency curves can be easily plotted.

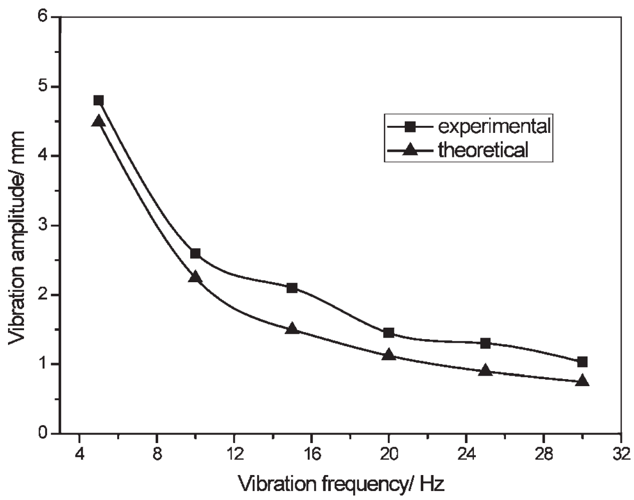

Figure 5 Comparison between theoretical and experimental amplitude–frequency characteristic curves of LDPE.

RESULTS AND DISCUSSIONS

To verify above proposed analytical model, the LDPE was selected as the experimental material to explore the relationship between vibration amplitude and vibration frequency in the electromagnetic dynamic tri-screw extruder. The experimental results of vibration amplitude–frequency characteristic obtained from above experiments are compared with the theoretical ones calculated with eq. (26) in Figure 5. It is clearly revealed that the theoretical and experimental results come to a good agreement. Although some deviation indeed exists, the mathematical derivations are feasible and reliable to predict the correlationship between the two vibration parameters when the testing and accidental errors are ignored. From Figure 5 we can further find that the experimental results are a little higher than the theoretical ones as a result of both the testing error and assumptions proposed when establishing the analytical model. From the comparison, it also can be revealed that the introduction of vibration force field in to polymer extrusion process can lower the viscosity of the polymer to some extents.

The nondimensionless amplitude–frequency characteristic curves (Fig. 6) are obtained from eq. (32).

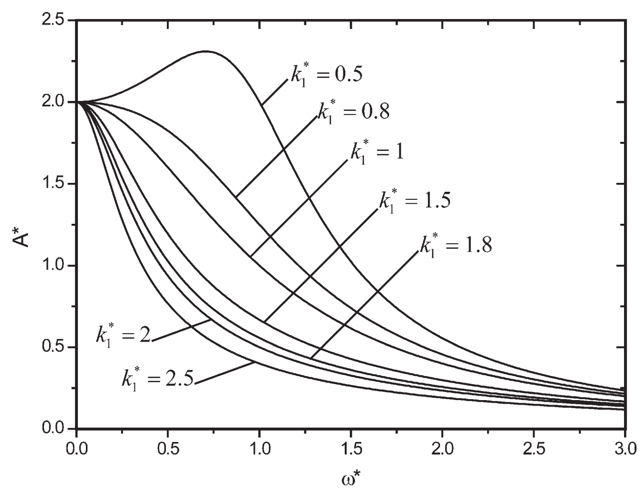

Figure 6 Amplitude–frequency characteristic curves.

The resonance of the exciting system happens at the wave crest ($k_1^* = 0.5$), however the wave crest disappears while $k_1^* \geq 1$, where the polymer melt viscosity is relatively high or proper parameters and extrusion technology is applied. Namely loaded resonant frequency of the system never exists and no response occurs. Actually, resonance will not take place within working frequency range, for axial vibration system turns to be over damping system under polymer melt damping force from high viscosity.

Besides, the dynamic magnification factor of amplitude with lower frequency exciting is higher than that of higher frequency exciting within working frequency range. The dynamic magnification factor of amplitude decreases with the increase of vibration frequency. The theoretical amplitude–frequency curves well interprets phenomenon in practical application. For instance, the vibration amplitude can only be adjusted within 0–2.5 mm at vibration frequency of 5 Hz, and that is limited within 0–1.5 mm at vibration frequency of 10 Hz. The adjustable amplitude range reduces to 0–1 mm at vibration frequency of 15 Hz. When the vibration frequency arrives at 20 Hz and 25 Hz, the adjustable amplitude range is limited within 0–0.5 mm.

CONCLUSIONS

Fluid mechanic method was applied herein to investigate the damping force and corresponding damping factor of electromagnetic dynamic tri-screw extruder. Forced vibration equations were established combining with exciting force term and finally the amplitude–frequency characteristic curves were obtained.

The possibility of resonance of the system is mainly determined by viscosity of raw material, geometry parameters of screws and processing techniques (rotating speed, temperature, pressure, etc.). Resonance of polymer in electromagnetic dynamic tri-screw extruder seldom occurs under normal extrusion conditions. Additionally, there exists an inverse relationship between vibration frequency and amplitude. The results play an important role in adjustments of processing technological parameters and design of polymer electromagnetic dynamic extrusion equipments. Besides, it offers reliable theoretical references for optimization of structure design for vibrating system of screw.

References

1. Fridman, M. L.; Peshkovsky, S. L.; Vinogradov, G. V. Polym Eng Sci 1981, 21, 755.
2. Sobhanie, M.; Isayevand, A. I.; Fan, Y. Rheol Acta 1997, 36, 66.
3. Qu, J.; Zeng, G.; Feng, Y.; Jin, G.; Heand, H.; Cao, X. J Appl Polym Sci 2006, 100, 3860.
4. Jin-ping, Q. U.S. Pat. 5,217,302 (1993).
5. Zeng, G. S.; Quand, J. P.; Feng, Y. H. J Appl Polym Sci 2007, 104, 2504.
6. Zengand, G.; Qu, J. J Appl Polym Sci 2007, 106, 1152.
7. Yue-Jun, L.; Xiang-Gangand, L.; Yu-Gang, H. The XV International Congress on Rheology: The Society of Rheology 80th Annual Meeting. 2008, 1027, 1192.
8. Lan, C. W. Int J Heat Mass Transfer 2000, 43, 1987.
9. Guangshengand, Z.; Jinping, Q. Polym Compos 2008, 29, 1252.
10. Jingping, P. X. Q. China Plast Ind 1999, 27, 41.